Innovative roof system for low-cost housing

Presentation

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Overview

1. Introduction
2. Aim
3. Cross-section: Roof
4. Data analysis
5. One-dimensional model
6. Final Remarks

Addressing the Need for Affordable Housing: Challenges

- The high cost of electricity and limited accessibility force people who live in informal settlements or shacks to opt for braziers as an alternative electric heaters to stay warm during the winter season.
- Climate change has made winters colder, forcing residents to leave the brazier running day and night for warmth.
- As such, braziers are one of the main causes of fire in these settlements during the winter season.
- There is a need to design housing that minimizes electricity consumption by keeping the house warm during winter. Ensure that the house remains cooler in summer.

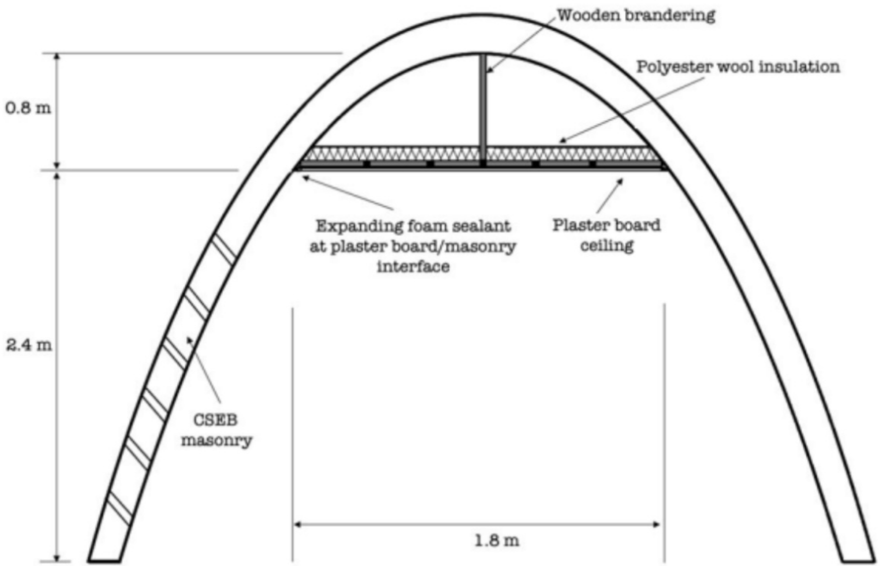
A brazier

The aim

- This proposed housing prototype is made up of recycled brick materials mixed with cement as a binding agent. It is a two-story house with a vaulted roof.
- Develop solutions to optimize the indoor thermal performance of the prototype, ensuring energy efficiency and comfort for residents in all seasons.

The housing prototype

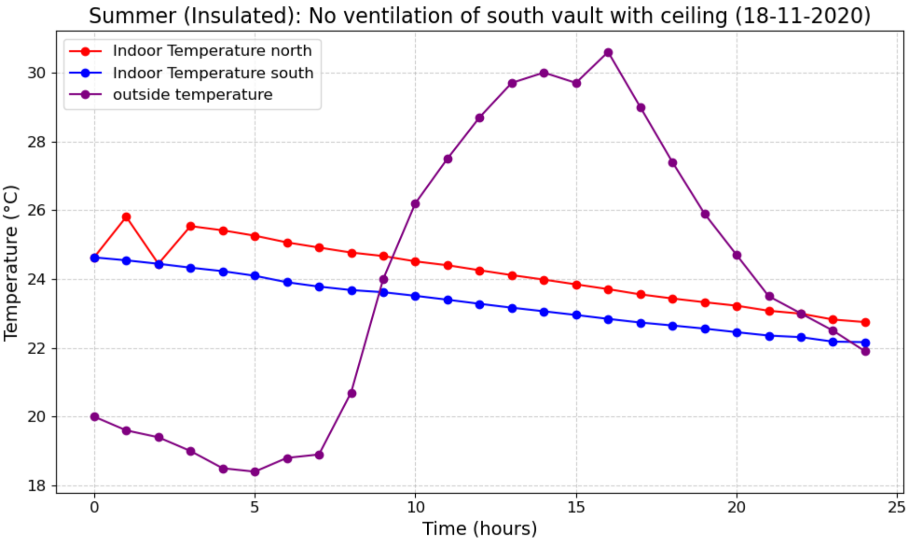
The housing prototype



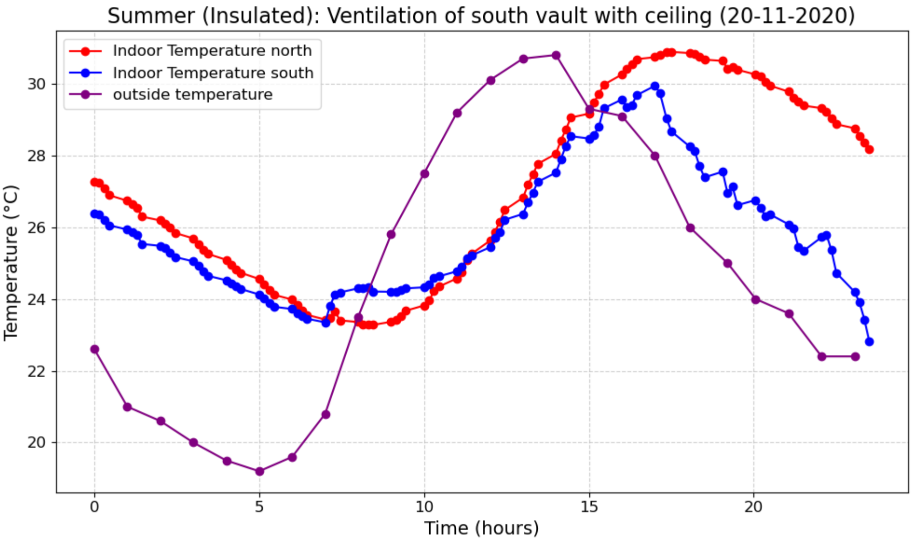
Data analysis: Time-series

- A glimpse of the obtained data describes the heat difference between indoors and outdoors. This includes cases when, windows are open, when the house is occupied, and the season.

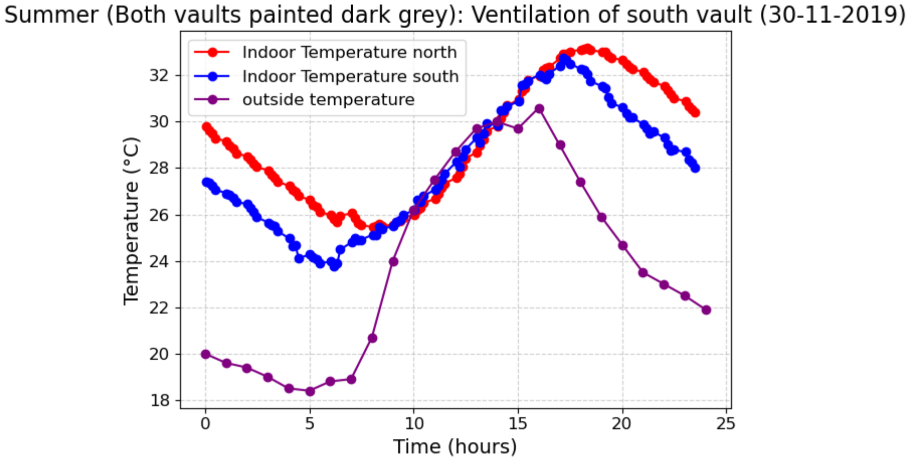
Data analysis: Time-series



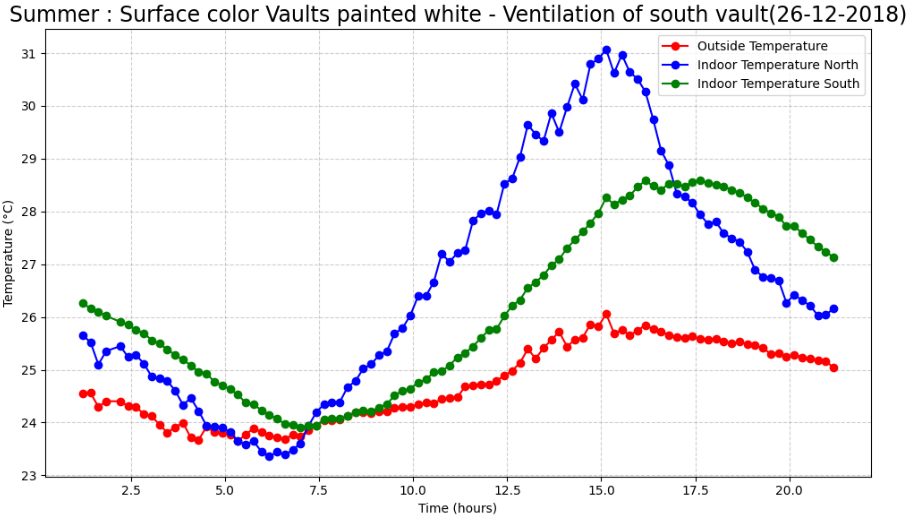
Data analysis: Time-series



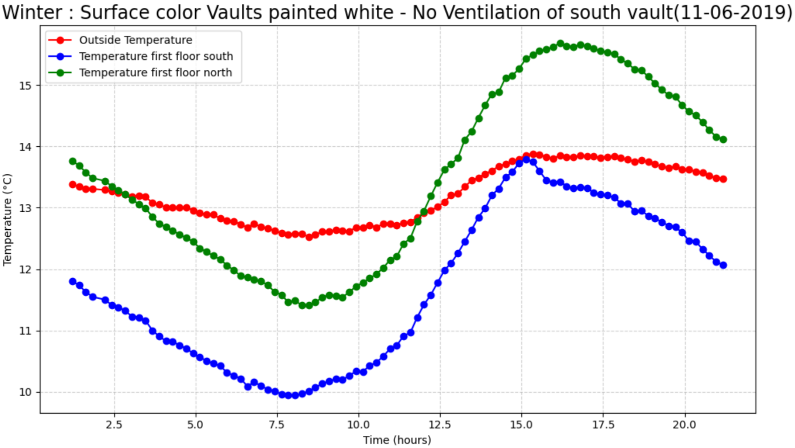
Data analysis: Time-series



Data analysis



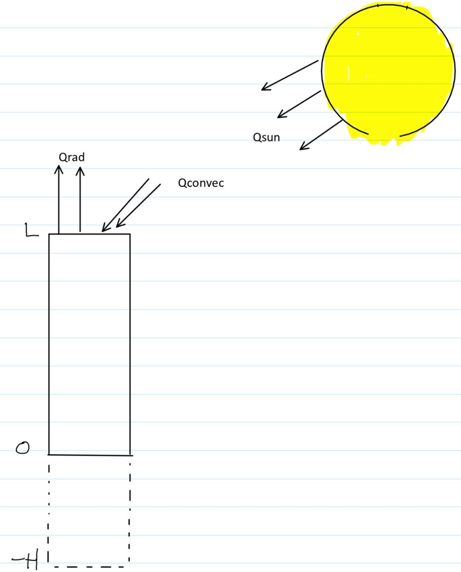
Data analysis



Data Summary

- The roof colour makes a difference to heat absorption, e.g. a white roof reflects a lot of heat, leaving a cooler room
- Keeping the windows closed in summer seems to provide a more stable environment.
- The delay between interior and exterior temperature is around 2 hours
- Performance in summer is beneficial to comfort, but in winter the great insulation seems to create a colder environment.
- There is a lot of data to be studied and analysed further.

One-dimensional Model



Equations and Boundary conditions

$$
\begin{aligned}
C_p \rho \frac{\partial u}{\partial t} &= \beta \frac{\partial^2 u}{\partial z^2} \\
-k \frac{\partial u}{\partial z}|_{z=L} &= Q_{sun} + Q_{rad} + Q_{conve}
\end{aligned}
$$

where $Q_{\text{sun}} = \gamma(1 - A) Q_{\text{max}} \theta(t), \quad Q_{\text{rad}} = \epsilon\sigma(u^4(t, L) - u_{amb}(t)^4),$
and $\quad Q_{\text{convec}} = \gamma\alpha_1(u(t, L) - u_{amb}(t))$

and on the underside of the roof,

$$
\frac{\partial u}{\partial z}|_{z=0} = -\frac{\alpha}{v}(u(0, t) - u_{air}(t)) + Q_{rad}
$$

v is related to the room volume, γ is a shape factor for the roof shape, u_{air} is the temperature in the room.

Equations and Boundary conditions

... and also to see what is happening to room temperature,

$$\frac{\partial u_{air}}{\partial t} = -\alpha[u(t,0) - u_{air}(t)] + Q_{rad} \tag{1}$$

Important parameters are; $\gamma = \textit{Shape factor}, A = \textit{Albedo}\ (\textit{Reflectivity})$
where γ is derived to consider the shape of the roof.

One-dimensional Model - shape factor

Consider the roof has shape $z = f(y)$. then the unit normal is;

$$\hat{\mathbf{n}} = \frac{\nabla(z - f(y))}{||\nabla(z - f(y))||} = \frac{-f'(y)\mathbf{j} + \mathbf{k}}{\sqrt{1 + f'(y)^2}}$$

The component of sunlight normal to the surface is then (the sun is directly above) then the sunlight normal to the surface $\hat{\mathbf{n}} \cdot \mathbf{k}$ integrated over the surface, i.e.

$$\int_S \frac{1}{\sqrt{1 + f'(y)^2}} \mathrm{d}s = \int_0^{2R} \mathrm{d}y,$$

since $\mathrm{d}s = \sqrt{1 + f'(y)^2}\ \mathrm{d}y$

- Sunlight is equivalent to that landing on a flat slab of length $2R$, where R is the half-width of the roof space.
- This heat must be distributed over the surface area of the roof itself.

$$\gamma = \frac{2Rl}{\text{Surface area of the roof}}$$

Shape Factor examples - reduction in roof heating

For a circular arched roof, where l is the length of the house, and the sun is passing lengthwise along the house, then

$$\gamma = \frac{2Rl}{\pi Rl} = \frac{2}{\pi}$$

while for a quadratic arch, the quantity would be

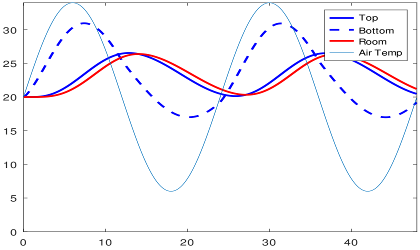
$$\gamma = \frac{2Rl}{2l\int_0^R (R^2 - y^2)\mathrm{d}y} = \frac{2Rl}{\frac{4}{3}R^3 l} = \frac{3}{2R^2}.$$

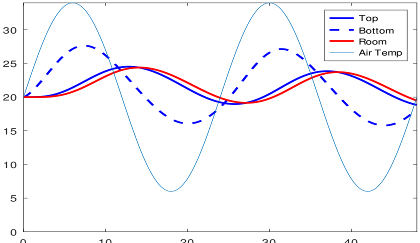
The total energy absorbed would be the same but it is distributed over a larger area.

Simulations

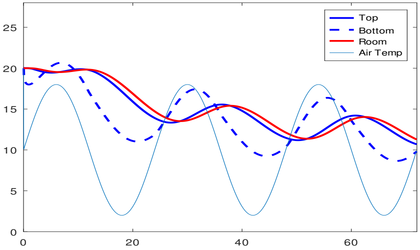
- Simulations conducted to test the model and effect of the shape factor.
- The difference between a domed roof and a flat roof
- Summer and winter simulation.
- Look to see if can you reproduce the situations in the data.
- Studies of impact of albedo and shape once the model is verified.

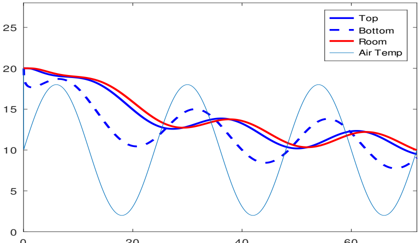
Hot Weather - Top: flat roof, bottom: quadratic roof





Cold Weather - Top: flat roof, bottom: quadratic roof





Future Model ideas - 1D

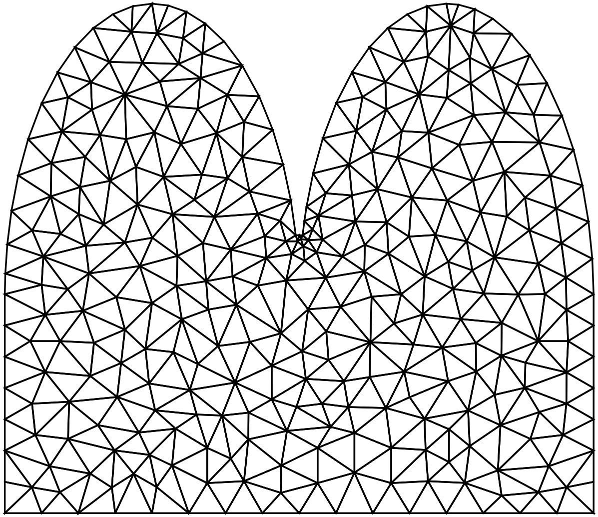
- Double check the parameters and model settings.
- Still some work to be done on this simple model to include different sun location and ventilation.
- Take two 1-D models side by side and include coupling between the two - to see differential heating from the two domes.

2D Model

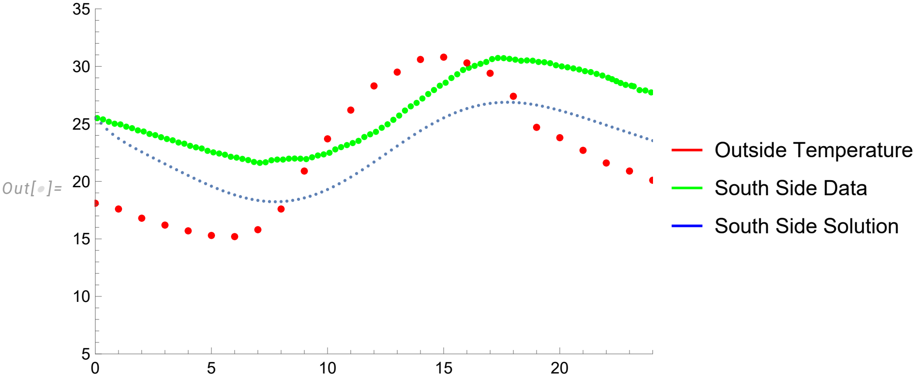
- Solved numerically using method of lines.
- Heavy lifting done mostly by Mathematica.
- Real outside temperature built into boundary condition

Mesh of Double Roof Building

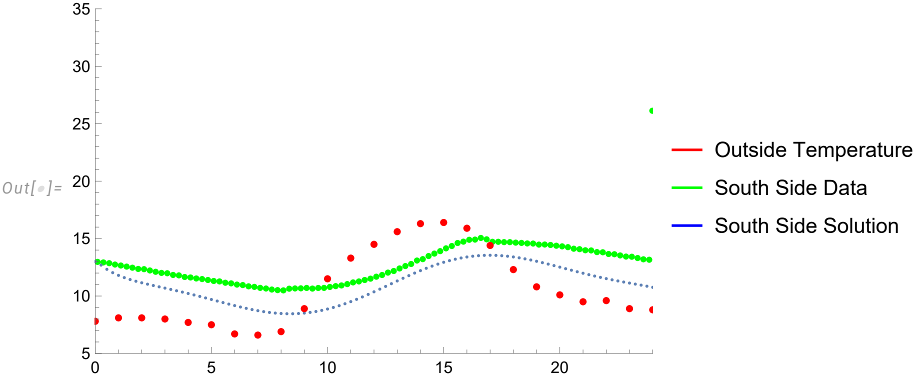
Out[•]=



Numerical Summer Solution



Numerical Winter Solution



Future Model ideas - 2D

- Model roof, walls and rooms as separate regions with their own properties.
- Include ventilation
- Divide Roof into two sections to account for shade and/or different heating boundary conditions

Conclusion and future work

- Examined the data to see what effects are evident.
- Have developed a 1D model of the heating/cooling in the structure depending on outdoor conditions/shape etc.
- A two-dimensional model using finite elements that builds in the shape but has some current limitations.
- Future work is to refine both models for ACTUAL conditions and with correct parameters, then investigate the scenarios.